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A novel diffusion process with jumps to study an electronic-optical edge router

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Abstract: The article presents a diffusion approximation model applied to investigate the process of filling a large optical packet by smaller and coming irregularly electronic packets. The use of diffusion approximation enables us to include the general distributions of interarrival times, also the self-similarity of the input process, as well as to investigate transient states. We propose a novel diffusion process with jumps representing the end of the filling the buffer due to arrival of too large packet and we give the transient solution to this process. The model allows us to study the distribution of interdeparture times and the distribution of the space occupied in the optical packet.
Keywords: diffusion approximation, all optical networks, self-similarity.

1. Introduction

Designing of smart edge routers, e.g. [9, 22, 27, 26], and shaping the self-similar traffic in optical switched networks [25] arise recently a lot of interest. Here, we propose an analytical approach which we consider useful in modelling and dimensioning of buffers in the edge routers between electronic and optical networks. We study a single buffer where packets of various sizes, classified by the class of service and the destination, are stored to build an optical packet of a fixed size. We already studied this problem with the use of simulation [24] model [8], remarking that self-similar traffic at the entrance of such a buffer remains self-similar when leaving it. Now we are building analytical model based on diffusion approximation. In section 2. we summarize the diffusion approximation and our previous contributions to this approach, i.e. a method to solve transient diffusion models, in section 3. we present the new model to analyse the process of buffer filling, in section 4. a numerical example proves that this approach may give reasonable results in relatively short (compared to simulation, especially simulation of transient states) time.

2. The principles of diffusion approximation: G/G/1/N model

Let $A(x)$, $B(x)$ denote the interarrival and service time distributions at a service station. The distributions are general, it is assumed that their two first moments are known: $E[A] = 1/\lambda$, $E[B] = 1/\mu$, $\text{Var}[A] = \sigma_A^2$, $\text{Var}[B] = \sigma_B^2$. Denote also the squared coefficients of variation $C_A^2 = \sigma_A^2\lambda^2$, $C_B^2 = \sigma_B^2\mu^2$. Let $N(t)$ be the number of customers present in the system at time t. For a single class FIFO queue, the changes $N(t + \Delta t) - N(t)$ have approximately normal distribution with mean $(\lambda - \mu)\Delta t$ and variance $(\sigma_A^2\lambda^3 + \sigma_B^2\mu^3)\Delta t$, provided that the time Δt is sufficiently long and the station is working without interruption, e.g. [20]. Diffusion approximation, e.g. [21, 15] replaces the process $N(t)$ by a continuous diffusion process $X(t)$ whose incremental changes $dX(t) = X(t + dt) - X(t)$ are normally distributed with the mean βdt and variance αdt, where β, α are the coefficients of the diffusion equation

$$\frac{\partial f(x,t;x_0)}{\partial t} = \frac{\alpha}{2}\frac{\partial^2 f(x,t;x_0)}{\partial x^2} - \beta\frac{\partial f(x,t;x_0)}{\partial x} \tag{1}$$

which defines the conditional pdf $f(x,t;x_0)dx = P[x \le X(t) < x + dx \mid X(0) = x_0]$ of $X(t)$. Hence, both processes $X(t)$ and $N(t)$ have normally distributed changes; the choice $\beta = \lambda - \mu$, $\alpha = \sigma_A^2\lambda^3 + \sigma_B^2\mu^3 = C_A^2\lambda + C_B^2\mu$ ensures the same ratio of time-growth of mean and variance of these distributions. Function $f(n,t;n_0)$ approximates the distribution $p(n,t;n_0)$ of customers of all classes present in the queue.

Formal justification of diffusion approximation is in limit theorems for $G/G/1$ system given by Iglehart [17]. If $\hat{N}_n$ is a series of random variables derived from $N(t)$:

$$\hat{N}_n = \frac{N(nt) - (\lambda - \mu)nt}{(\sigma_A^2\lambda^3 + \sigma_B^2\mu^3)\sqrt{n}}\,,$$

then the series is weakly convergent (in the sense of distribution) to ξ where $\xi(t)$ is a standard Wiener process (diffusion process with $\beta = 0$ i $\alpha = 1$) provided that $\varrho > 1$, i.e. if the system is unstable. In the case of $\varrho = 1$ the series $\hat{N}_n$ is convergent to ξ_R. The $\xi_R(t)$ process is $\xi(t)$ process limited to half-axis $x > 0$: $\xi_R(t) = \xi(t) - \inf\,[\xi(u),\ \ 0 \le u \le t]$. There is no similar theorems for service stations in equilibrium ($\varrho < 1$) and we should rely on heuristic confirmation of the utility of this approximation.

If the input stream λ is composed of K classes of customers having intensities $\lambda^{(k)}$, with total intensity $\lambda = \sum_{k=1}^{K}\lambda^{(k)}$ and service parameters for a class k are

$E[B^{(k)}] = 1/\mu^k$, Var$[B^{(k)}] = {\sigma_B^{(k)}}^2$, then the PDF $B(x)$ of joint for all classes service time distribution is expressed as

$$B(x) = \sum_{k=1}^{K} \frac{\lambda^{(k)}}{\lambda} B^{(k)}(x) \ ,$$

and

$$\frac{1}{\mu} = \sum_{k=1}^{K} \frac{\lambda^{(k)}}{\lambda} \frac{1}{\mu^{(k)}} \ , \qquad C_B^2 = \mu^2 \sum_{k=1}^{K} \frac{\lambda_{(k)}}{\lambda} \frac{1}{{\mu^{(k)}}^2} ({C_B^{(k)}}^2 + 1) - 1 \ . \tag{2}$$

If we assume that the input streams are independent, the global number of arrived during Δt customers is normally distributed with variance $\lambda C_A^2 \Delta t = \sum_{k=1}^{K} \lambda^{(k)} {C_A^{(k)}}^2 \Delta t$, hence

$$C_A^2 = \sum_{k=1}^{K} \frac{\lambda^{(k)}}{\lambda} {C_A^{(k)}}^2 \ . \tag{3}$$

The above equations yield α, β of the diffusion equation.

Boundary conditions for Eq. (1) should be also defined. In [15] diffusion approximation of a $G/G/1/N$ station was studied as a process $X(t)$ which is defined on the closed interval $x \in [0, N]$. When the process comes to $x = 0$, it remains there for a time exponentially distributed with the parameter λ and then it returns to $x = 1$; when it comes to $x = N$, it remains there for a time which is exponentially distributed with the parameter μ and then it starts at $x = N - 1$. The use of barriers with jumps (instantaneous returns) gives better results than the use of reflecting barriers applied earlier [21] where probability of the process being at the barriers was neglected. With barriers with jumps, the diffusion equation is supplemented by the balance equations for $p_0(t) = P[X(t) = 0]$, $p_N(t) = P[X(t) = N]$ and becomes

$$\begin{aligned}
\frac{\partial f(x,t;x_0)}{\partial t} &= \frac{\alpha}{2} \frac{\partial^2 f(x,t;x_0)}{\partial x^2} - \beta \frac{\partial f(x,t;x_0)}{\partial x} + \lambda p_0(t) \delta(x-1) \\
&\quad + \mu p_N(t) \delta(x - N + 1) \ , \\
\frac{dp_0(t)}{dt} &= \lim_{x \to 0} [\frac{\alpha}{2} \frac{\partial f(x,t;x_0)}{\partial x} - \beta f(x,t;x_0)] - \lambda p_0(t) \ , \\
\frac{dp_N(t)}{dt} &= - \lim_{x \to N} [\frac{\alpha}{2} \frac{\partial f(x,t;x_0)}{\partial x} - \beta f(x,t;x_0)] - \mu p_N(t) \ .
\end{aligned} \tag{4}$$

In original works [15, 16] only the steady-state solution of Eq. (4) is given. Our approach, proposed in [4], is first to solve the diffusion equation with absorbing barriers (the process ends when it comes to a barrier) placed at $x = 0$ and $x = N$ with the use of standard analytical methods (the method of mirrors) and obtain the pdf $\phi(x,t;x_0)$

of this process, then to express the pdf $f(x,t;x_0)$ of the diffusion with instantaneous returns from the barriers as a superposition of functions $\phi(x,t;x_0)$:

$$\begin{aligned} f(x,t;\psi) &= \phi(x,t;\psi) + \int_0^t g_1(\tau)\phi(x,t-\tau;1)d\tau + \\ & \int_0^t g_{N-1}(\tau)\phi(x,t-\tau;N-1)d\tau \end{aligned} \tag{5}$$

where for $t=0$, $\phi(x,t;x_0) = \delta(x-x_0)$ and for $t>0$

$$\begin{aligned} \phi(x,t;x_0) &= \frac{1}{\sqrt{2\Pi\alpha t}} \sum_{n=-\infty}^{\infty} \Big\{ \exp\left[\frac{\beta x'_n}{\alpha} - \frac{(x-x_0-x'_n-\beta t)^2}{2\alpha t}\right] - \\ & \exp\left[\frac{\beta x''_n}{\alpha} - \frac{(x-x_0-x''_n-\beta t)^2}{2\alpha t}\right]\Big\}, \end{aligned} \tag{6}$$

where $x'_n = 2nN$, $x''_n = -2x_0 - x'_n$, and ψ is the initial condition and $\phi(x,t;\psi) = \int_0^N \phi(x,t;\xi)\psi(\xi)d\xi$. Functions $g_1(\tau)$, $g_{N-1}(\tau)$ are the densities of starting new processes (after a jump from the barrier) at points $x=1$ and $x=N-1$. These densities are given by a system of balance equations (7), (8) for probability flows coming *in* and *out* of the barriers; the equations make use of first passage times from starting points to the barriers and of the densities of sojourn times in the barriers (the assumption on exponentially distributed times spent in barriers is not now needed).

Densities $\gamma_0(t)$, $\gamma_N(t)$ of probability that at time t the process enters $x=0$ or $x=N$ are

$$\begin{aligned} \gamma_0(t) &= p_0(0)\delta(t) + [1-p_0(0)-p_N(0)]\gamma_{\psi,0}(t) + \\ & \int_0^t g_1(\tau)\gamma_{1,0}(t-\tau)d\tau + \int_0^t g_{N-1}(\tau)\gamma_{N-1,0}(t-\tau)d\tau \ , \\ \gamma_N(t) &= p_N(0)\delta(t) + [1-p_0(0)-p_N(0)]\gamma_{\psi,N}(t) + \\ & \int_0^t g_1(\tau)\gamma_{1,N}(t-\tau)d\tau + \int_0^t g_{N-1}(\tau)\gamma_{N-1,N}(t-\tau)d\tau \ , \end{aligned}$$

where $\gamma_{1,0}(t)$, $\gamma_{1,N}(t)$, $\gamma_{N-1,0}(t)$, $\gamma_{N-1,N}(t)$ are densities of the first passage times between corresponding points, e.g.

$$\gamma_{1,0}(t) = \lim_{x\to 0}[\frac{\alpha}{2}\frac{\partial\phi(x,t;1)}{\partial x} - \beta\phi(x,t;1)] \ . \tag{7}$$

For absorbing barriers

$$\lim_{x\to 0}\phi(x,t;x_0) = \lim_{x\to N}\phi(x,t;x_0) = 0 \ ,$$

hence $\gamma_{1,0}(t) = \lim_{x\to 0} \frac{\alpha}{2}\frac{\partial\phi(x,t;1)}{\partial x}$. The density function of first passage time from $x = x_0$ to $x = 0$ is

$$\gamma_{x_0,0}(t) = \lim_{x\to 0}[\frac{\alpha}{2}\frac{\partial}{\partial x}\phi(x,t;x_0) - \beta\phi(x,t;x_0)] = \frac{x_0}{\sqrt{2\Pi\alpha t^3}}e^{-\frac{(\beta t+1)^2}{2\alpha t}} .$$

The functions $\gamma_{\psi,0}(t)$, $\gamma_{\psi,N}(t)$ denote densities of probabilities that the initial process, started at $t = 0$ at the point ξ with density $\psi(\xi)$ will end at time t by entering respectively $x = 0$ or $x = N$.

Densities $g_1(t)$ and $g_N(t)$ may be expressed with the use of functions $\gamma_0(t)$ and $\gamma_N(t)$:

$$g_1(\tau) = \int_0^\tau \gamma_0(t)l_0(\tau - t)dt \ , \qquad g_{N-1}(\tau) = \int_0^\tau \gamma_N(t)l_N(\tau - t)dt \ , \quad (8)$$

where $l_0(x)$, $l_N(x)$ are the densities of sojourn times in $x = 0$ and $x = N$; the distributions of these times are not restricted to exponential ones as it is in Eq. (4). The integrals in Eq. (5) are in fact convolutions of functions and we may rewrite this equation as

$$f(x,t;\psi) = \phi(x,t;\psi) + g_1(t) * \phi(x,t;1)d\tau + g_{N-1}(t) * \phi(x,t;N-1) \quad (9)$$

where * denotes the convolution, or, transforming it with the use of Laplace transform, as

$$\bar{f}(x,s;\psi) = \bar{\phi}(x,s;\psi) + \bar{g}_1(s)\,\bar{\phi}(x,s;1) + \bar{g}_{N-1}(s)\,\bar{\phi}(x,s;N-1) \, . \quad (10)$$

Laplace transforms of Eqs. (7,8) give $\bar{g}_1(s)$ and $\bar{g}_{N-1}(s)$, hence the Laplace transform $\bar{f}(x,s;\psi)$ of the density function $f(x,t;\psi)$ is obtained and supplemented by transforms of probabilities that at the moment t the process is in a barrier

$$\bar{p}_0(s) = \frac{1}{s}\left[\bar{\gamma}_0(s) - \bar{g}_1(s)\right] , \qquad \bar{p}_N(s) = \frac{1}{s}\left[\bar{\gamma}_N(s) - \bar{g}_{N-1}(s)\right] . \quad (11)$$

Expressions (10), (11) are inverted numerically, e.g. with the use of Stehfest algorithm [23].

This transient solution is obtained for constant parameters. To introduce $\alpha(t)$, $\beta(t)$ reflecting evolution of input streams, the time axis is divided into small intervals during which parameters are kept constant and the solution at the end of one interval gives the initial conditions to the diffusion equation in the next interval and with new parameters. Sometimes we need diffusion parameters $\alpha(x,t)$, $\beta(x,t)$ depending also on the value of the process – this is a way to reflect control mechanisms reacting on the queue size or to model parallel servers. In this case, the diffusion interval $x \in [0,N]$ is divided into subintervals of appropriate (e.g. unitary) length and the parameters are kept constant within these subintervals. The equations for space-intervals are solved together with balance equations for probability flows between neighbouring intervals. For each time- and space-subinterval with constant parameters, transient solution is obtained. As previously, the Laplace transforms of density functions are inverted numerically.

This method was implemented in a software package [12] and is able to solve large queueing network models: in [14] we analyzed transient states at a network of 37 stations of G/G/20/20 type representing a part of wireless cellular network. It was also used to model dynamics of flows subject to some traffic control mechanisms encountered in communication networks:

— control mechanisms at the entrance of a network: leaky bucket, also in presence of correlated (self-similar) input [11], jumping window [1], sliding window [18].

— space-priority queues at a network switch: a queue with threshold [10] and with push-out algorithm [3];

— the traffic dynamics along a virtual path composed of a certain number of nodes [5], and a feed-back algorithm of traffic control between nodes and sources [1, 2].

3. The buffer model

Basing on the solution developed in the previous section and using similar notation, we build a diffusion model for the assembly buffer at electronic-optical (E/O) node. We assume that the size of incoming electronic packets is between 1 and M blocks, without specifying the size of block, i.e. the granularity of the input stream. The input stream in the model is composed of M independent streams and a stream m, $m = 1, \dots M$ represents packets of the size of m blocks. The parameters of this stream are λ_m, C^2_{Am}. As the diffusion process represents the number of blocks, we should determine the mean value and variance of the number of blocks arriving during a time-unit. Assuming that once for m times we have an arrival of a block following specified pattern and then $(m-1)$ immediate arrivals correlated with the first one, we obtain the parameters for the block interarrival time distribution

$$\lambda_{mb} = m\lambda_m, \qquad C^2_{Amb} = mC^2_{Am} + m - 1. \tag{12}$$

The parameters of the total input stream being a sum of M streams are computed using (3) and (12). We study the accumulation of blocks in the buffer, therefore the diffusion parameters are defined only by input process, there is no service time,

$$\beta = \sum_{m=1}^{M} \lambda_{mb}, \qquad \alpha = \sum_{m=1}^{M} \lambda_{mb} C^2_{Amb}. \tag{13}$$

We assume the asynchronous work of the node: when an incoming packet is too large to be put into the buffer (the number of blocks in this packet is greater than the place still available in the buffer), the content of the buffer is sent as optical packet and the last packet that did not matched the left buffer space is put to the empty buffer – the process of building a new packet is being started. We assume also the time-out T after which the packet is sent regardless its content.

The goal of the model is to obtain the distribution of interdeparture times and the distribution of the number of blocks occupied in the optical packet leaving the node.

Let N be the size of optical buffer expressed in blocks. The value of the diffusion process $X(t)$ at time t represents the current number of blocks already occupied inside the buffer. We consider diffusion process on the interval $x \in (0, N-1)$. Within this interval we distinguish M subintervals: subinterval number 1 $(0, N-M)$, and $M-1$ subintervals of unitary length: subinterval number 2 is $x \in (N-M, N-M+1)$, $\ldots$, subinterval number M is $x \in (N-2, N-1)$. Between subintervals, i.e. at the points $x = N-M$, $N-M+1$, $\ldots$, $N-2$ and $x = N-1$ we place imaginary barriers that allow us to make a balance of probability flows and to represent interactions among subintervals. The barriers are absorbing ones, as at previous section, hence we may use for each subinterval the solution for the process density $f(x, t; x_0)$ similar to the obtained above in Eq. (9) giving only proper starting points and starting intensities. The flows absorbed by the barriers reappear on their other side at the distance ε.

Let $\gamma_i^L(t)$ represent the flow coming to the barrier placed at $x = i$ from its left side and $\gamma_i^R(t)$ be the flow coming to this barrier from its right side. The flows start diffusion processes at both sides of the barrier, at points $x = i - \varepsilon$ and $x = i + \varepsilon$ with intensities $g_i^L(t)$ and $g_i^R(t)$. The whole flow $\gamma_i^R(t)$ is transmitted to $x = i - \varepsilon$: $g_{N-i}^L(t) = \gamma_{N-i}^R(t)$, but only a part of the flow $g_i^L(t)$ enters $x = i + \varepsilon$. The flow $g_i^L(t)$ is divided in the following way. The part of this probability flow which corresponds to the arrivals of packets which are smaller then $M - i$ blocks and may be put into the buffer reappears immediately at $x = i + \varepsilon$ as $g_i^R(t)$. The part of $\gamma_i^L(t)$ which represents the arrivals of packets having exactly the size of $M - i$ blocks still available at the buffer is directed to the barrier at $x = 0$ (the optical packet is sent full, with maximal number of blocks occupied). The part of $\gamma_i^L(t)$ representing the flow of packets of size $k > N - i$ which are too large to be stocked in the currant optical packet is directed to the points $x = k$ at the first interval. The barrier at $x = N - 1$ injects flows to $x = 0$ (the arrivals of one-block packets completing the buffer) and to $x = 2, \ldots, M$.

The barrier at $x = 0$ acts similarly as in $G/G/1/N$ model, the sojourn time in this barrier corresponds to the time when the buffer is empty, then the jumps are performed to points $x = 1, \ldots M$ as to the empty buffer the packets of the size $1, \ldots, M$ may arrive. The circulation of probability mass is illustrated in figs. 1, 2. Having all this in mind we determine the intensity $g_{0,k}(t)$ of jumps (i.e the density of starting a new diffusion process at a corresponding point) from the barrier at $x = 0$ to a point $x = k$:

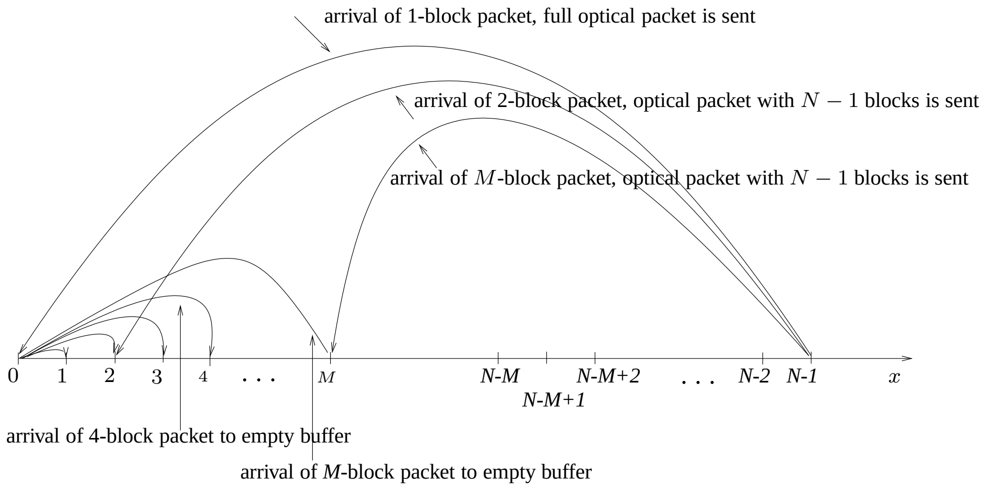

Fig. 1. Jumps from barriers at both ends of the interval

$$g_{0,k}(\tau) = \int_0^\tau \frac{k\lambda_k}{\sum_{l=1}^M l\lambda_l} \gamma_0(t) l_0(\tau - t) dt \ ,$$

where $l_0(t)$ is the sojourn time density and the input flow $\gamma_0(t)$ contains all flows directed to the barrier from other barriers as well as flows coming to if from starting points inside the first interval:

$$\gamma_0(t) = \gamma_{\psi_1,0}(t) + \sum_{l=1}^M g_l(t) * \gamma_{l,0}(t) + g_{N-M}^L(t) * \gamma_{N-M-\varepsilon}(t) + \sum_{l=1}^M \gamma_{N-l}^L(t)\Lambda_l.$$

The first term corresponds to the flow coming from initial distribution of the probability mass (function $\psi_1(x)$ inside first interval), the second represents flows coming from starting points inside first interval ($x = 1, 2, \ldots, M$ and $x = N - M - \varepsilon$), and the last sum gathers flows from barriers. We denote:

$$\Lambda_i = \frac{i\lambda_i}{\sum_{l=1}^M l\lambda_l}, \qquad \Omega_k = \frac{\sum_{l=1}^k \Lambda_l}{\sum_{l=1}^M \Lambda_l}.$$

Let us note that

$$g_{N-i}^R(t) = \Omega_{i-1}\gamma_{N-i}^L(t) \qquad \text{and} \qquad g_i(t) = \Lambda_i \gamma_0(t) * l_0(t).$$

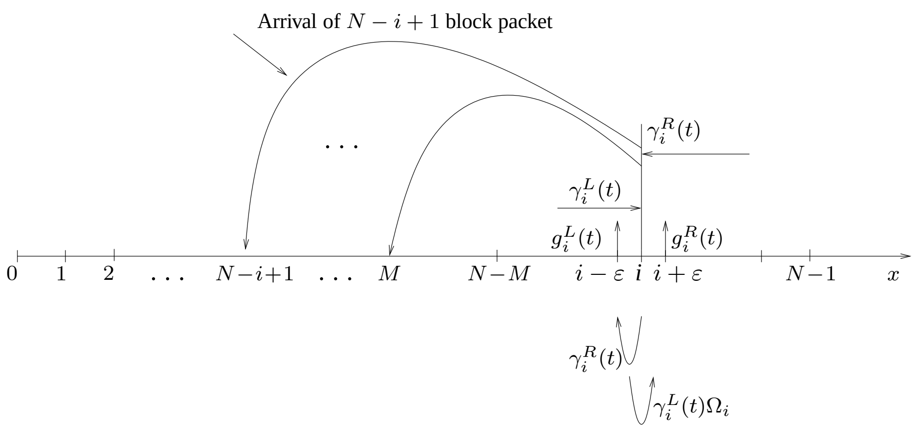

Fig. 2. Jumps from intermediate barriers

The solution for the first subinterval $0 < x \le N - M$

$$f_1(x,t;\psi_1) = \phi(x,t;\psi_1) + \sum_{i=1}^{M} g_i(t) * \phi(x,t;i) + + g^L_{N-M}(t) * \phi(x,t;N-M-\varepsilon), \quad (14)$$

for an interval $i = 2, \ldots M - 1$

$$f_i(x,t;\psi_i) = \phi(x,t;\psi_i) + g^R_{N-M+i-2}(t) * \phi(x,t;N-M+i-2+\varepsilon) + + g^L_{N-M+i-1}(t) * \phi(x,t;N-M+i-1-\varepsilon), \quad (15)$$

and

$$f_M(x,t;\psi_M) = \phi(x,t;\psi_i) + g^R_{N-2}(t) * \phi(x,t;N-2+\varepsilon)(t) \quad (16)$$

We obtain steady-state solution finding

$$\lim_{s\to 0} s\bar{f}(x,s;\psi_i) = \lim_{t\to\infty} f(x,t;\psi_i).$$

The density of the packet interdeparture times $d(t)$ is obtained by summing all densities that end the buffer filling, namely all probability flows from barriers at $x = N - M, \ldots N - 1$ to points $x = 0, 1, \ldots, M$ computed in the model where these flows are not reinjected into the diffusion interval but accumulated in a supplementary state "departure of the packet".

To incorporate in the model the timeout T, the probability mass which at time $t = T$ is still inside the interval $(0, N-1)$ (have not yet gone to the supplementary state) is at this moment moved immediately to the supplementary state. The distribution of the number of blocs inside the dispatched packet is obtained using probability of all possible events: probability mass accumulated at the supplementary state through the jumps from barriers to $x = 0$ represents probability that the packet leaves with all blocks occupied. Probability mass accumulated in the supplementary state coming through jumps to $x = 1$ represents probability that the packet leaves with one block empty, etc.

4. Numerical example

In numerical example below we take $N = 100$ or 200, $M = 20$, $p_m = 1/M$ (the traffic intensity for each stream is $\lambda_m = 0.01$), $m = 1, \ldots 20$, i.e. the size of optical packet is $N = 100$ or 200 blocks, the electronic packets are of size $1, \ldots, 20$ blocks, and all these packets are equiprobable. The streams are Poisson, hence $C^2_{Am} = 1$, $m = 1, \ldots 20$. Naturally, we may easily insert any value of λ_m, p_m, C^2_{Am}.

Figs. 3, 4 show the solution $f(x, t; 0)$ of diffusion equations, that means the functions $f_1(x, t; 0)$, …, $f_M(x, t; 0)$ given by eqs. (14), (15), (16). They present the buffer filling as a function of time. We have chosen the case $x_0 = 0$: the initial buffer is empty. Numerical inversion of Laplace transforms was done with the use of Stehfest algorithm [23]. In this algorithm a function $f(t)$ is obtained from its transform $\bar{f}(s)$ for any fixed argument t as

$$f(t) = \frac{\ln 2}{2} \sum_{i=1}^{N} V_i \, \bar{f}\left(\frac{\ln 2}{t} i\right) , \tag{17}$$

where

$$V_i = (-1)^{N/2+i} \times \sum_{k=\lfloor \frac{i+1}{2} \rfloor}^{\min(i,N/2)} \frac{k^{N/2+1}(2k)!}{(N/2-k)!k!(k-1)!(i-k)!(2k-i)!}. \tag{18}$$

N is an even integer and depends on a computer precision; we used $N = 14$.

5. Conclusions

Presented model may be useful to investigate the transient states and the dynamics of flows on the edge between electronic and optical networks. They may include self-similar input rates [7] and investigate the influence of the node on the characteristics of the traffic. We do not present here any real validation of the model, but a comparison with simulation results presented in [8] as well as long-term experience with other diffusion models, and their validation via simulations, presented e.g. in [3, 4, 5, 1, 2, 11, 10, 13, 12, 19, 14, 7], prove that such models give in general reasonable estimations. However, the programming effort needed to obtain them is not negligible.

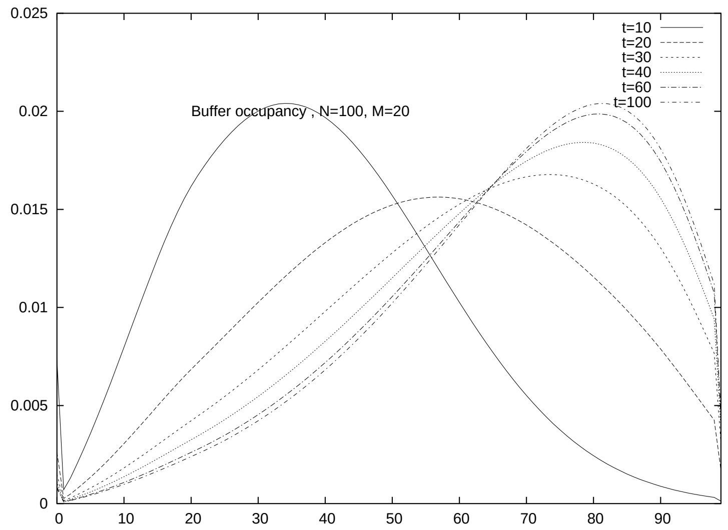

Fig. 3: Distribution $f(x,t;0)$ of the number of blocks in the buffer as a function of time, $N = 100$.

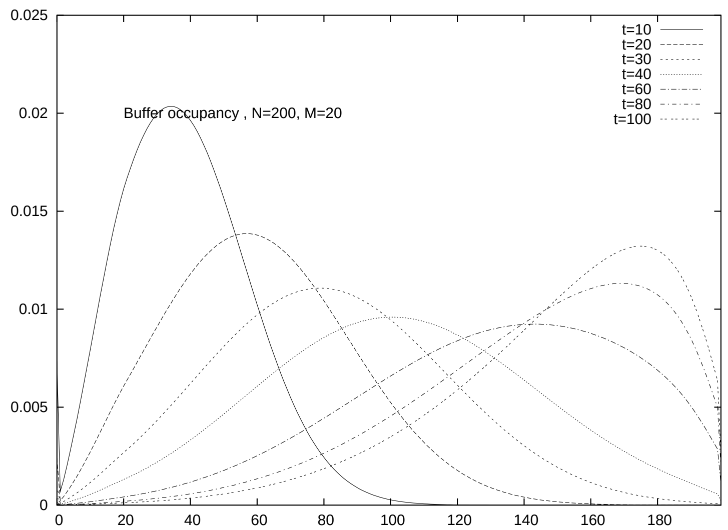

Fig. 4: Distribution $f(x,t;0)$ of the number of blocks in the buffer as a function of time, $N = 200$.

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